Light-cone QCD Sum Rules for the Λ Baryon Electromagnetic Form Factors and its magnetic moment

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We present the light-cone QCD sum rules up to twist 6 for the electromagnetic form factors of the Λ baryon. To estimate the magnetic moment of the baryon, the magnetic form factor is fitted by the dipole formula. The numerical value of our estimation is $\mu_\Lambda = -(0.64 \pm 0.04)\mu_N$, which is in accordance with the experimental data and the existing theoretical results. We find that it is twist 4 but not the leading twist distribution amplitudes that dominate the results.

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I. INTRODUCTION

Electromagnetic (EM) form factors of the hadron are fundamental objects, for they characterize the internal structure of the composite particle. There were a lot of experimental results on EM form factors of baryons [1, 2, 3, 4] and mesons [5, 6, 7, 8, 9, 10] in the past decades. Due to the complexity of their structure, theoretical investigations on baryons received much less attention than those on mesons, and the existing studies on EM form factors of baryons were mainly focused on the nucleon. In comparison with the nucleon, so far as we know, there are no experimental data for Λ at present. Therefore, it is instructive and necessary to study its EM form factors theoretically. Fortunately, there have been experimental result on the magnetic moment and other model-dependent theoretical results on EM form factors to test our estimation indirectly. Van Cauteren *et al.* have investigated the electric and magnetic form factors of strange baryons in the relativistic constituent-quark model [11]. The chiral perturbation theory [12] and the chiral quark/soliton model [13] have been used to investigate the Λ EM form factors at low momentum transfer. The present work is devoted to investigate the Λ EM form factors at moderately large momentum transfer and estimate its magnetic moment theoretically.

The matrix element of the EM current between the initial and final Λ baryon states can be parameterized in terms of the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$:

$$\langle \Lambda(P')|j_\mu^{em}(0)|\Lambda(P)\rangle = \bar{\Lambda}(P')[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2M_\Lambda}F_2(Q^2)]\Lambda(P), \qquad (1)$$

where $j_\mu^{em} = e_u\bar{u}\gamma_\mu u + e_d\bar{d}\gamma_\mu d + e_s\bar{s}\gamma_\mu s$ is the EM current relevant to the baryon. P and P' are the four-momenta of the initial and final Λ baryon states, M_Λ is the Λ mass, $P^2 = P'^2 = M_\Lambda^2$, $Q^2 = -q^2 = -(P - P')^2$ is the momentum transfer (outgoing photon

momentum), and $\Lambda(P)$ is the Λ spinor. From the experimental point of view, Dirac and Pauli form factors are described equivalently by another set of form factors: electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ Sachs form factors:

$$
\begin{aligned}
G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2), \\
G_M(Q^2) &= F_1(Q^2) + F_2(Q^2).
\end{aligned}
\tag{2}
$$

The normalization of the Sachs form factors at $Q^2 = 0$ is given by the baryon charge $G_E(0) = 0$ and magnetic moment $G_M(0) = \mu_\Lambda$. In the Breit frame, $G_E(Q^2)$ corresponds to the distribution of the electric charge and $G_M(Q^2)$ to the magnetic current distribution.

To give the EM form factors, QCD sum rule method from a three-point correlation function is a useful tool. However, the three-point sum rule has a deficiency which restricts its application and accuracy [14]. Instead of it, the light-cone sum rule (LCSR) method, which has firstly been employed on the nucleon by Braun et $al.$ [15], is utilized to study the Λ EM form factors in this paper. LCSR, a hybrid of the traditional sum rule (SVZ sum rule) [16] and the theory of hard exclusive processes, is a useful method to include both hard scattering and soft contributions in QCD. It was developed in late 1980's by making a partial resummation of the operator product expansion (OPE) to all orders [17, 18, 19]. The main difference between SVZ sum rule and LCSR is that the short-distance Wilson OPE in increasing dimension is replaced by the light-cone expansion in terms of distribution amplitudes (see Ref. [20] for a review) of increasing twist. The Λ baryon distribution amplitudes, which are the fundamental input parameters in LCSR, have been given in Ref. [21, 22]. In this paper, we give a little correction to the nonperturbative parameter λ_1 which has different sign compared with that in Ref. [22]. To leading order in the QCD coupling α_s, we give the Q^2-dependence EM form factors of Λ in the range $1\,\mathrm{GeV}^2 \leq Q^2 \leq 7\,\mathrm{GeV}^2$ considering contributions up to twist 6.

Another aim of this manuscript is to estimate the Λ magnetic moment. As important intrinsic physical values, the magnetic moments of baryons have been investigated theoretically in the past years by various models [12, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. Among these models, QCD sum rule is widely used. The conventional way to calculate the magnetic moments of hadrons in QCD sum rule is to deal with the two-point correlation function in the background of the electromagnetic field at zero momentum transfer. This is the developed QCD sum rule method proposed by Balitsky et $al.$ [34] and Ioffe et $al.$ [35]. In their method, the correlation function is expanded in a constant weak external electromagnetic field $F_{\mu\nu}$, and the sum rule of the term connected with the magnetic moment can be gotten directly. This method has been widely used in calculating the magnetic moments of hadrons [23, 34, 35, 36, 37, 38, 39, 40]. In the present paper, another approach is adopted to evaluate the Λ magnetic moment. The basic idea of the approach is to extract the magnetic moment from the calculated EM form factors indirectly. It is assumed that the magnetic form factor divided by the magnetic moment can be described by the dipole

formula:

$$\frac{1}{\mu_\Lambda}G_M(Q^2) = \frac{1}{(1+Q^2/m_0^2)^2} = G_D(Q^2). \tag{3}$$

Since there are no experimental data available for the form factor, the result is fitted by the above dipole formula, and the two parameters μ_Λ and m_0^2 are artificial parameters to be determined simultaneously. After that, we use the fit to estimate the magnetic moment of the baryon. In Ref. [32], the authors have ever calculated the magnetic moment of the Λ baryon with LCSR. In their calculations, the magnetic moment can be obtained directly. This comes from the fact that they use the photon distribution amplitudes, which allows for the utilization of LCSR at the point $Q^2 = 0$. However, in our case, the working region of the sum rule cannot extrapolate to the zero point directly. Hence, we have to estimate it with other approaches.

This paper is organized as follows. Section II is devoted to derive the light-cone QCD sum rules of the form factors relevant to the momentum transfer Q^2, and the necessary distribution amplitudes needed in the calculation are presented in this section. Section III is the numerical analysis part in which the magnetic form factor is fitted by the dipole formula, and the magnetic moment is estimated from the fit. This section also presents the QCD sum rules of the nonperturbative parameters f_Λ and λ_1. The summary and conclusion are given at the end of this section.

II. LIGHT-CONE QCD SUM RULES FOR THE EM FORM FACTORS

A. Leading twist

The derivation of the sum rules begins with the following correlation function:

$$T_\mu(P, q) = i \int d^4x e^{iq \cdot x} \langle 0|T\{j_\Lambda(0)j_\mu^{em}(x)\}|\Lambda(P)\rangle, \tag{4}$$

where the interpolating current of the Λ baryon field is chosen to be

$$j_\Lambda(0) = \epsilon_{ijk}(u^i(0)C\gamma_5 \not{z}d^j(0)) \not{z}s^k(0). \tag{5}$$

The coupling constant is defined by the matrix element of the interpolating current between the vacuum and the Λ state:

$$\langle 0|j_\Lambda|\Lambda\rangle = f_\Lambda(P \cdot z) \not{z}\Lambda(P), \tag{6}$$

where z is a light-cone vector, $z^2 = 0$, and f_Λ determines the normalization of the leading twist Λ distribution amplitude.

In order to get the sum rules, the correlation function (4) needs to be expressed both phenomenologically and theoretically. On one side, by inserting a complete set of interme- diate states with the same quantum numbers as those of Λ, and using the definition of the

form factors in Eq. (1) and the definition in Eq. (6), the hadronic representation of the correlation function is expressed as follows:

$$
\begin{aligned}
z^\mu T_\mu(P,q) &= \frac{1}{M_\Lambda^2 - P'^2} f_\Lambda(P' \cdot z)[2(P' \cdot z F_1(Q^2) \\
&\quad - \frac{q \cdot z}{2} F_2(Q^2)) \not{z} + (P' \cdot z F_2(Q^2) \\
&\quad + \frac{q \cdot z}{2} F_2(Q^2)) \frac{\not{z} \not{q}}{M_\Lambda}]\Lambda(P) + ... \, ,
\end{aligned}
\tag{7}
$$

where $P' = P - q$, and the dots stand for the higher resonance and continuum contributions. Here the contraction of the correlation function by z^μ is adopted to get rid of contributions which are subdominant on the light cone.

On the other side, the correlation function is calculated in perturbation theory at large Euclidean momenta P'^2 and $q^2 = -Q^2$ in terms of the distribution amplitudes. To the leading order, the theoretical side of the correlation function is

$$
\begin{aligned}
z^\mu T_\mu(P,q) &= 2(P \cdot z)^2 \not{z}\Lambda(P) \int \mathcal{D}\alpha \{e_u \frac{\mathcal{A}_1(\alpha)}{s_1 - (q-P)^2} \\
&\quad + e_d \frac{\mathcal{A}_1(\alpha)}{s_2 - (q-P)^2} + e_s \frac{\mathcal{A}_1(\alpha)}{s_3 - (q-P)^2}\},
\end{aligned}
\tag{8}
$$

where $s_i = (1-\alpha_i)P^2 - (1-\alpha_i)/\alpha_i q^2 + m_i^2/\alpha_i$, and $m_{1,2} = 0, m_3 = m_s$. The Λ baryon distribution amplitudes are defined by the matrix element of the operator between vacuum and the Λ state. The leading order distribution amplitudes are expressed as

$$
\begin{aligned}
&4\langle 0|u_\alpha^i(a_1 x)d_\beta^j(a_2 x)s_\gamma^k(a_3 x)|\Lambda(P)\rangle \\
&= \mathcal{V}_1(\not{P}C)_{\alpha\beta}(\gamma_5\Lambda)_\gamma + \mathcal{A}_1(\not{P}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma \\
&\quad + \mathcal{T}_1(P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Lambda)_\gamma.
\end{aligned}
\tag{9}
$$

The invariant functions $\mathcal{V}_1, \mathcal{A}_1$ and $\mathcal{T}_1$ can be represented explicitly:

$$
F(a_i p \cdot x) = \int \mathcal{D}\alpha e^{-ip \cdot x \Sigma_i \alpha_i a_i} F(\alpha_i) \, .
\tag{10}
$$

The integration measure is defined as follows:

$$
\int \mathcal{D}\alpha = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1).
\tag{11}
$$

As the usual procedure in QCD sum rule, by taking into account the dispersion relation and the quark-hadron duality, the hadronic representation of the correlation function is matched with that calculated on the light cone. Then after making the Borel transformation to suppress higher resonance contributions, we get the following light-cone QCD sum

rules:

$$f_\Lambda F_1(Q^2)e^{-\frac{M_\Lambda^2}{M_B^2}} = \{e_u \int_{\alpha_{1_0}}^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \mathcal{A}_1(\alpha_1,\alpha_2,1-\alpha_1-\alpha_2)e^{-\frac{s_1}{M_B^2}}$$

$$+e_d \int_{\alpha_{2_0}}^1 d\alpha_2 \int_0^{1-\alpha_2} d\alpha_1 \mathcal{A}_1(\alpha_1,\alpha_2,1-\alpha_1-\alpha_2)e^{-\frac{s_2}{M_B^2}}$$

$$+e_s \int_{\alpha_{3_0}}^1 d\alpha_3 \int_0^{1-\alpha_3} d\alpha_1 \mathcal{A}_1(\alpha_1,1-\alpha_1-\alpha_3,\alpha_3)e^{-\frac{s_3}{M_B^2}}\},$$

$$F_2(Q^2) = 0. \tag{12}$$

Here α_{i_0} connects with the continuum threshold s_0:

$$\alpha_{i_0} = \frac{\sqrt{(s_0+Q^2-M_\Lambda^2)^2+4(Q^2+m_i^2)M_\Lambda^2}}{2M_\Lambda^2} - \frac{(s_0+Q^2-M_\Lambda^2)}{2M_\Lambda^2} \tag{13}$$

B. Beyond the leading twist

It is known that for the nucleon, the leading twist contribution is rather small, while higher twist contributions are dominant [15], which is different from cases of mesons. Therefore, it is necessary to consider contributions of the higher twist distribution amplitudes for the calculation of the Λ EM form factors. The usual higher twist contributions come from two different physical origins. First, as the hard quark propagator will receive corrections when considering the background gluon field, there may come contributions from the four-particle (and five-particle) distribution amplitudes. Second, contributions will arise from the matrix element of the three-quark operator in Eq. (9) if we consider other Lorentz structures on the light cone [22, 41, 42]. As the first source does not play a significant role [15, 43], we only consider the second one in this paper.

In the calculation, only axial-like vector Lorentz structures contribute, so we merely present the following decomposition of the matrix element of the three-quark operator:

$$4\langle 0|\epsilon^{ijk}u_\alpha^i(a_1x)d_\beta^j(a_2x)s_\gamma^k(a_3x)|\Lambda(P)\rangle$$
$$= \mathcal{A}_1(\slashed{P}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_2 M(\slashed{P}\gamma_5 C)_{\alpha\beta}(\slashed{x}\Lambda)_\gamma + \mathcal{A}_3 M(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\Lambda)_\gamma$$
$$+\mathcal{A}_4 M^2(\slashed{x}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_5 M^2(\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu\Lambda)_\gamma + \mathcal{A}_6 M^3(\slashed{x}\gamma_5 C)_{\alpha\beta}(\slashed{x}\Lambda)_\gamma. \tag{14}$$

As the above distributions do not have a definite twist (see Ref. [15, 42] for a review), the invariant functions $\mathcal{A}_i$ can be parameterized in terms of distribution amplitudes $A_1,\ldots,A_6$

with a definite twist:

$$
\begin{aligned}
\mathcal{A}_1 &= A_1, \\
2P \cdot x \mathcal{A}_2 &= -A_1 + A_2 - A_3, \\
2\mathcal{A}_3 &= A_3, \\
4P \cdot x \mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5, \\
4P \cdot x \mathcal{A}_5 &= A_3 - A_4, \\
(2P \cdot x)^2 \mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6,
\end{aligned}
\tag{15}
$$

where A_1 is twist-3, A_2 and A_3 are twist-4, A_4 and A_5 are twist-5, and A_6 is twist-6 distribution amplitude.

These distribution amplitudes are scale dependent and can be expanded into contributions of conformal operators. To the leading order conformal spin accuracy the expansion reads [22, 42]

$$
\begin{aligned}
A_1(x_i, \mu) &= -120 x_1 x_2 x_3 \phi_3^0(\mu), \\
A_2(x_i, \mu) &= -24 x_1 x_2 \phi_4^0(\mu), \\
A_3(x_i, \mu) &= -12 x_3 (1 - x_3) \psi_4^0(\mu), \\
A_4(x_i, \mu) &= -3(1 - x_3) \psi_5^0(\mu), \\
A_5(x_i, \mu) &= -6 x_3 \phi_5^0(\mu), \\
A_6(x_i, \mu) &= -2 \phi_6^0(\mu).
\end{aligned}
\tag{16}
$$

With the equation of motion, the six parameters can be expressed in terms of two independent parameters f_Λ and λ_1. To the leading order conformal spin accuracy, they are expressed as

$$
\begin{aligned}
\phi_3^0 &= \phi_6^0 = -f_\Lambda, \\
\phi_4^0 &= \phi_5^0 = -\frac{1}{2}(f_\Lambda + \lambda_1), \\
\psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_\Lambda - \lambda_1).
\end{aligned}
\tag{17}
$$

Considering the definition of Eq. (14) and Eq. (15), the correlation function (4) is calculated up to twist 6 and described explicitly:

$$
\begin{aligned}
z^\mu T_\mu(P, q) &= 2 e_u (P \cdot z)^2 \, \slashed{z} \Lambda(P) \int_0^1 d\alpha_1 \Big\{ B_0(\alpha_1) + B_1(\alpha_1) \frac{M_\Lambda^2}{(s_1 - (q-P)^2)} \\
&+ B_2(\alpha_1) \frac{2 M_\Lambda^4}{(s_1 - (q-P)^2)^2} \Big\} \frac{1}{s_1 - (q-P)^2} + 2 e_u (P \cdot z)^2 \, \slashed{z} \slashed{q} \Lambda(P) \\
&\times \int_0^1 d\alpha_1 \Big\{ E_1(\alpha_1) - B_2(\alpha_1) \frac{2 M_\Lambda^2}{\alpha_1 (s_1 - (q-P)^2)^2} \Big\} \frac{M_\Lambda}{\alpha_1 (s_1 - (q-P)^2)^2} \\
&+ (e_u \rightarrow e_d, \alpha_1 \leftrightarrow \alpha_2, s_1 \rightarrow s_2, B_i \rightarrow C_i, E_1 \rightarrow E_2) \\
&+ (e_u \rightarrow e_s, \alpha_1 \rightarrow \alpha_3, \alpha_2 \rightarrow \alpha_1, s_1 \rightarrow s_3, B_i \rightarrow D_i, E_1 \rightarrow E_3).
\end{aligned}
\tag{18}
$$

Here the following notation is used for convenience:

$$
\begin{aligned}
B_0(\alpha_1) &= \int_0^{1-\alpha_1} d\alpha_2 A_1(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2), \\
B_1(\alpha_1) &= 2\widetilde{A}_1(\alpha_1) - \widetilde{A}_2(\alpha_1) + \widetilde{A}_3(\alpha_1) + \widetilde{A}_4(\alpha_1) - \widetilde{A}_5(\alpha_1), \\
B_2(\alpha_1) &= \widetilde{\widetilde{A}}_1(\alpha_1) - \widetilde{\widetilde{A}}_2(\alpha_1) + \widetilde{\widetilde{A}}_3(\alpha_1) + \widetilde{\widetilde{A}}_4(\alpha_1) - \widetilde{\widetilde{A}}_5(\alpha_1) + \widetilde{\widetilde{A}}_6(\alpha_1), \\
C_0(\alpha_2) &= \int_0^{1-\alpha_2} d\alpha_1 A_1(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2), \\
C_1(\alpha_2) &= 2\widetilde{A}_1(\alpha_2) - \widetilde{A}_2(\alpha_2) + \widetilde{A}_3(\alpha_2) + \widetilde{A}_4(\alpha_2) - \widetilde{A}_5(\alpha_2), \\
C_2(\alpha_2) &= \widetilde{\widetilde{A}}_1(\alpha_2) - \widetilde{\widetilde{A}}_2(\alpha_2) + \widetilde{\widetilde{A}}_3(\alpha_2) + \widetilde{\widetilde{A}}_4(\alpha_2) - \widetilde{\widetilde{A}}_5(\alpha_2) + \widetilde{\widetilde{A}}_6(\alpha_2), \\
D_0(\alpha_3) &= \int_0^{1-\alpha_3} d\alpha_1 A_1(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3), \\
D_1(\alpha_3) &= 2\widetilde{A}_1(\alpha_3) - \widetilde{A}_2(\alpha_3) + \widetilde{A}_3(\alpha_3) + \widetilde{A}_4(\alpha_3) - \widetilde{A}_5(\alpha_3), \\
D_2(\alpha_3) &= \widetilde{\widetilde{A}}_1(\alpha_3) - \widetilde{\widetilde{A}}_2(\alpha_3) + \widetilde{\widetilde{A}}_3(\alpha_3) + \widetilde{\widetilde{A}}_4(\alpha_3) - \widetilde{\widetilde{A}}_5(\alpha_3) + \widetilde{\widetilde{A}}_6(\alpha_3), \\
E_1(\alpha_1) &= -\widetilde{A}_1(\alpha_1) + \widetilde{A}_2(\alpha_1) - \widetilde{A}_3(\alpha_1), \\
E_2(\alpha_2) &= -\widetilde{A}_1(\alpha_2) + \widetilde{A}_2(\alpha_2) - \widetilde{A}_3(\alpha_2), \\
E_3(\alpha_3) &= -\widetilde{A}_1(\alpha_3) + \widetilde{A}_2(\alpha_3) - \widetilde{A}_3(\alpha_3),
\end{aligned}
\tag{19}
$$

where the distribution amplitudes with a "tilde" are defined as

$$
\begin{aligned}
\widetilde{A}(\alpha_1) &= \int_0^{\alpha_1} d\alpha_1' \int_0^{1-\alpha_1'} d\alpha_2 A(\alpha_1', \alpha_2, 1 - \alpha_1' - \alpha_2), \\
\widetilde{A}(\alpha_2) &= \int_0^{\alpha_2} d\alpha_2' \int_0^{1-\alpha_2'} d\alpha_1 A(\alpha_1, \alpha_2', 1 - \alpha_1 - \alpha_2'), \\
\widetilde{A}(\alpha_3) &= \int_0^{\alpha_3} d\alpha_3' \int_0^{1-\alpha_3'} d\alpha_1 A(\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'), \\
\widetilde{\widetilde{A}}(\alpha_1) &= \int_0^{\alpha_1} d\alpha_1' \int_0^{\alpha_1'} d\alpha_1'' \int_0^{1-\alpha_1''} d\alpha_2 A(\alpha_1'', \alpha_2, 1 - \alpha_1'' - \alpha_2), \\
\widetilde{\widetilde{A}}(\alpha_2) &= \int_0^{\alpha_2} d\alpha_2' \int_0^{\alpha_2'} d\alpha_2'' \int_0^{1-\alpha_2''} d\alpha_1 A(\alpha_1, \alpha_2'', 1 - \alpha_1 - \alpha_2''), \\
\widetilde{\widetilde{A}}(\alpha_3) &= \int_0^{\alpha_3} d\alpha_3' \int_0^{\alpha_3'} d\alpha_3'' \int_0^{1-\alpha_3''} d\alpha_1 A(\alpha_1, 1 - \alpha_1 - \alpha_3'', \alpha_3'').
\end{aligned}
\tag{20}
$$

These results stem from the partial integration in α_1, α_2 and α_3, respectively, which is employed to eliminate the factors $1/(P \cdot x)^n$ in the calculation.

Now by using the Borel transformation and subtraction similarly to that in Ref. [15],

we arrive at the final sum rules:

$$f_\Lambda F_1(Q^2)e^{-\frac{M_\Lambda^2}{M_B^2}} = e_u\Big\{ \int_{\alpha_{1_0}}^1 d\alpha_1\{B_0(\alpha_1) + B_1(\alpha_1)\frac{M_\Lambda^2}{M_B^2} + B_2(\alpha_1)\frac{M_\Lambda^4}{M_B^4}\}e^{-\frac{s_1}{M_B^2}} + \{B_1(\alpha_{1_0})$$

$$+B_2(\alpha_{1_0})\frac{M_\Lambda^2}{M_B^2} - \frac{d}{d\alpha_{1_0}}\frac{\alpha_{1_0}^2 M_\Lambda^2 B_2(\alpha_{1_0})}{Q^2 + \alpha_{1_0}^2 M_\Lambda^2}\}\frac{M_\Lambda^2\alpha_{1_0}^2}{Q^2 + \alpha_{1_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}$$

$$+e_d\Big\{ \int_{\alpha_{2_0}}^1 d\alpha_2\{C_0(\alpha_2) + C_1(\alpha_2)\frac{M_\Lambda^2}{M_B^2} + C_2(\alpha_2)\frac{M_\Lambda^4}{M_B^4}\}e^{-\frac{s_2}{M_B^2}}$$

$$+\{C_1(\alpha_{2_0}) + C_2(\alpha_{2_0})\frac{M_\Lambda^2}{M_B^2} - \frac{d}{d\alpha_{2_0}}\frac{\alpha_{2_0}^2 M_\Lambda^2 C_2(\alpha_{2_0})}{Q^2 + \alpha_{2_0}^2 M_\Lambda^2}\}\frac{M_\Lambda^2\alpha_{2_0}^2}{Q^2 + \alpha_{2_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}$$

$$+e_s\Big\{ \int_{\alpha_{3_0}}^1 d\alpha_3\{D_0(\alpha_3) + D_1(\alpha_3)\frac{M_\Lambda^2}{M_B^2} + D_2(\alpha_3)\frac{M_\Lambda^4}{M_B^4}\}e^{-\frac{s_3}{M_B^2}} + \{D_1(\alpha_{3_0})$$

$$+D_2(\alpha_{3_0})\frac{M_\Lambda^2}{M_B^2} - \frac{d}{d\alpha_{3_0}}\frac{\alpha_{3_0}^2 M_\Lambda^2 D_2(\alpha_{3_0})}{Q^2 + m_s^2 + \alpha_{3_0}^2 M_\Lambda^2}\}\frac{M_\Lambda^2\alpha_{3_0}^2}{Q^2 + m_s^2 + \alpha_{3_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}, \tag{21}$$

and

$$f_\Lambda F_2(Q^2)e^{-\frac{M_\Lambda^2}{M_B^2}} = 2\Big\{e_u\Big\{ \int_{\alpha_{1_0}}^1 d\alpha_1\{\frac{M_\Lambda^2}{\alpha_1 M_B^2}E_1(\alpha_1) - \frac{M_\Lambda^4}{\alpha_1 M_B^4}B_2(\alpha_1)\}e^{-\frac{s_1}{M_B^2}}$$

$$+\{\frac{1}{\alpha_{1_0}}E_1(\alpha_{1_0}) - \frac{M_\Lambda^2}{\alpha_{1_0} M_B^2}B_2(\alpha_{1_0}) + \frac{d}{d\alpha_{1_0}}\frac{\alpha_{1_0} M_\Lambda^2}{Q^2 + \alpha_{1_0}^2 M_\Lambda^2}B_2(\alpha_{1_0})\}$$

$$\times\frac{\alpha_{1_0}^2 M_\Lambda^2}{Q^2 + \alpha_{1_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}$$

$$+e_d\Big\{ \int_{\alpha_{2_0}}^1 d\alpha_2\{\frac{M_\Lambda^2}{\alpha_2 M_B^2}E_2(\alpha_2) - \frac{M_\Lambda^4}{\alpha_2 M_B^4}C_2(\alpha_2)\}e^{-\frac{s_2}{M_B^2}}$$

$$+\{\frac{1}{\alpha_{2_0}}E_2(\alpha_{2_0}) - \frac{M_\Lambda^2}{\alpha_{2_0} M_B^2}C_2(\alpha_{2_0})$$

$$+\frac{d}{d\alpha_{2_0}}\frac{\alpha_{2_0} M_\Lambda^2}{Q^2 + \alpha_{2_0}^2 M_\Lambda^2}C_2(\alpha_{2_0})\}\frac{\alpha_{2_0}^2 M_\Lambda^2}{Q^2 + \alpha_{2_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}$$

$$+e_s\Big\{ \int_{\alpha_{3_0}}^1 d\alpha_3\{\frac{M_\Lambda^2}{\alpha_3 M_B^2}E_3(\alpha_3) - \frac{M_\Lambda^4}{\alpha_3 M_B^4}D_2(\alpha_3)\}e^{-\frac{s_3}{M_B^2}}$$

$$+\{\frac{1}{\alpha_{3_0}}E_3(\alpha_{3_0}) - \frac{M_\Lambda^2}{\alpha_{3_0} M_B^2}D_2(\alpha_{3_0}) + \frac{d}{d\alpha_{3_0}}\frac{\alpha_{3_0} M_\Lambda^2}{Q^2 + m_s^2 + \alpha_{3_0}^2 M_\Lambda^2}$$

$$\times D_2(\alpha_{3_0})\}\frac{\alpha_{3_0}^2 M_\Lambda^2}{Q^2 + m_s^2 + \alpha_{3_0}^2 M_\Lambda^2}e^{-\frac{s_0}{M_B^2}}\Big\}\Big\}. \tag{22}$$

III.   NUMERICAL ANALYSIS

A.   Determination of the parameters f_Λ and λ_1

The parameters f_Λ and λ_1 can be determined by the QCD sum rule method. The sum rules begin with the following correlation functions:

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j_i(x)\bar{j}_i(0)\}|0\rangle, \tag{23}$$

where the interpolating currents are

$$\begin{aligned} j_1(x) &= \epsilon_{ijk}(u^i(0)C\gamma_5 \slashed{z} d^j(0))\slashed{z}s^k(0), \\ j_2(x) &= \epsilon_{ijk}(u^i(x)C\gamma_5\gamma_\mu d^j(x))\gamma_\mu s^k(x), \end{aligned} \tag{24}$$

and following the standard QCD sum rule procedure, the results are given by

$$(4\pi)^4 f_\Lambda^2 e^{-\frac{M_\Lambda^2}{M_B^2}} = \frac{2}{5}\int_{m_s^2}^{s_0} s(1-x)^5 e^{-\frac{s}{M_B^2}} ds - \frac{b}{3}\int_{m_s^2}^{s_0} x(1-x)(1-2x)e^{-\frac{s}{M_B^2}}\frac{ds}{s},$$

$$\begin{aligned} 4(2\pi)^4 \lambda_1^2 M_\Lambda^2 e^{-\frac{M_\Lambda^2}{M_B^2}} &= \int_{m_s^2}^{s_0} \frac{s^2}{2}[(1-x^2)(1-8x+x^2) - 12x^2\ln(x)]e^{-\frac{s}{M_B^2}} ds \\ &\quad + \frac{b}{12}\int_{m_s^2}^{s_0}(1-x)^2 e^{-\frac{s}{M_B^2}} ds - \frac{4}{3}a^2 e^{-\frac{m_s^2}{M_B^2}} + m_s a_s \int_{m_s^2}^{s_0} e^{\frac{s}{M_B^2}} ds, \end{aligned} \tag{25}$$

where $x = m_s^2/s$.

The sum rules reveal that they can only give the absolute values of the two parameters. The relative sign of f_Λ and λ_1 can be determined by the following correlation function:

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j_1(x)\bar{j}_2(0)\}|0\rangle, \tag{26}$$

from which the sum rule of $f_\Lambda \lambda_1^*$ is given as follows:

$$\begin{aligned} 4(2\pi)^4 f_\Lambda \lambda_1^* M_\Lambda e^{-\frac{M_\Lambda^2}{M_B^2}} &= \int_{m_s^2}^{s_0} \frac{m_s}{6} s[(1-x)(3+13x-5x^2+x^3)+12x\ln(x)]e^{-\frac{s}{M_B^2}} ds \\ &\quad + \frac{b}{12}\int_{m_s^2}^{s_0} m_s(1-x)[1 - \frac{1}{3}(1-x)(5-\frac{2}{x})]e^{-\frac{s}{M_B^2}}\frac{ds}{s} \\ &\quad + \frac{a_s}{24}\int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds. \end{aligned} \tag{27}$$

In the numerical analysis we adopt the standard values:

$$\begin{aligned} a &= -(2\pi)^2\langle\bar{q}q\rangle = 0.55 \text{ GeV}^3, \\ b &= (2\pi)^2\langle\alpha_s G^2/\pi\rangle = 0.47 \text{ GeV}^4, \\ a_s &= m_0^2 a, \quad m_0^2 = 0.8 \text{ GeV}^2. \end{aligned} \tag{28}$$

The threshold is set to be $s_0 = 1.6^2$ GeV^2, and the working window for the Borel parameter is 1 GeV$^2 < M_B^2 < 2$ GeV^2. The sum rule (27) shows that $f_\Lambda \lambda_1^*$ is positive. Here f_Λ is taken to be positive and the parameters have the following numerical values:

$$
\begin{aligned}
f_\Lambda &= (5.9 \pm 0.2) \times 10^{-3} \text{ GeV}^2, \\
\lambda_1 &= (1.0 \pm 0.3) \times 10^{-2} \text{ GeV}^2.
\end{aligned}
\tag{29}
$$

B. Analysis of the sum rules

Before the numerical analysis, we firstly specify the input parameters used in the light-cone sum rules. The mass of the Λ baryon is given by the Particle Data Group (PDG) [44]: $M_\Lambda = 1.116$ GeV. The mass of the strange quark is chosen to be $m_s = 0.15$ GeV, and the continuum threshold $s_0 = 2.45 - 2.65$ GeV^2. For the auxiliary Borel parameter M_B^2, a working window in which the results vary mildly is required. The choice of the Borel parameter should satisfy two conditions simultaneously. On the one hand, as the higher twist contributions are proportional to terms $(1/M_B^2)^n$ ($n = 1, 2, ...$), M_B^2 should be large enough to suppress the higher twist contributions. On the other hand, M_B^2 cannot be too large in case that higher resonance and continuum contributions become dominant. In the calculation, the Borel parameter varies in the range 2 GeV$^2 \leq M_B^2 \leq 4$ GeV^2. Fig. 1(a) gives the dependence of the magnetic form factors on the Borel parameter at different point of Q^2. One can see from the figure that the results are almost independent of the Borel parameter.

In the following numerical analysis of the sum rules the Borel parameter is taken to be $M_B^2 = 3$ GeV^2, and the nonperturbative parameters f_Λ and λ_1 are the central values in Eq. (29). Fig. 1(b) gives the dependence of the magnetic form factor $G_M(Q^2)$ on the momentum transfer Q^2. The figure shows that the Q^2-dependence of the magnetic form factor $G_M(Q^2)$ is in accordance with our assumption in Eq. (3). In order to estimate the magnetic moment, the magnetic form factor is fitted by the dipole formula (3). The fits are shown in Fig. 2, from which we evaluate the numerical values of the two parameters simultaneously: $\mu_\Lambda = -(0.64 \pm 0.04)\mu_N$ and $m_0^2 = (0.89 \pm 0.04)$ GeV^2. In Fig. 3(a) we also display the Q^2-dependence of the value $G_M/(\mu_\Lambda G_D)$. The parameter m_0^2 used in the dipole formula (3) is the central value obtained above, while the magnetic moment $\mu_\Lambda = 0.613\mu_N$ comes from Ref. [44]. It is shown from the figure that the result deviates from 1 when Q^2 becomes large, which is due to the fact that the absolute value of $G_M(Q^2)$ form factor decreases with Q^2, and the influence of the threshold s_0 becomes important at large momentum transfer.

Tab. I lists results of Λ magnetic moment from various approaches. In comparison with results in Tab. I, our estimation is in accordance with data provided by PDG and other theoretical predictions.

Fig. 3(b) is the Q^2-dependence of the electric form factor $G_E(Q^2)$. The figure shows

that the electric form factor changes sign at a finite value of Q^2, which means that in the large momentum transfer the massive s quark play a more important role in determining the electric density of the baryon. This is different from that of the neutron [15]. A similar conclusion has been given in Ref. [11]. The difference to our calculation is that the electric form factor is negative at lower Q^2 in their calculation, while another result from Ref. [13] within the frame-work of the chiral quark/soliton model showed that at small Q^2 this form factor is positive, which is in contradiction to the result from Ref. [11].

Finally, contributions of distribution amplitudes with different twist are calculated for $G_M(Q^2)$, which is shown in Fig. 4. It can be concluded that it is twist 4 but not leading twist contributions that dominate the form factor. This stems from the structure of the leading order distribution amplitude of the Λ baryon, which is symmetric on the d and s quark if the mass of s quark approaches zero. Contributions of d and s quark can be canceled by the contribution of u quark in the approach. The results are expected to be better if more information about the distribution amplitudes of the Λ baryon is known.

To summarize, we provide a fit approach to predict the magnetic moment of a hadron. The Q^2-dependence EM form factors of the Λ baryon are calculated in the framework of the light-cone sum rule up to twist 6. The magnetic form factor is fitted by the dipole formula to estimate the magnetic moment of the baryon. Our estimation is in accordance with the existing results. As there lack experimental data on the baryon, we only give theoretical investigations on the form factors as what have been done on nucleons. Analysis on the electric form factor shows that it changes sign at a finite large momentum transfer, which is expected to be tested by the future experiments. Studies on contributions of the distribution amplitudes with different twist show that it is the twist 4 but not the leading twist contributions that dominate the result.

Acknowledgments

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Figure and table captions

Fig. 1.(a) Dependence of the Λ magnetic form factor $G_M(Q^2)$ on the Borel parameter. The lines correspond to the points $Q^2 = 1,\ 2,\ 3,\ 5\ \mathrm{GeV}^2$ from the bottom up with the threshold $s_0 = 2.55\ \mathrm{GeV}^2$. (b) Q^2-dependence of the magnetic form factor G_M. The lines correspond to the threshold $s_0 = 2.45 - 2.65\ \mathrm{GeV}^2$ from up down.

Fig. 2. Fittings of the form factor $G_M(Q^2)$ by $\mu_\Lambda/(1 + Q^2/m_0^2)^2$ where the dashed lines are the fits. Figures (a), (b) correspond to threshold $s_0 = 2.45,\ 2.65\ \mathrm{GeV}^2$, respectively.

Fig. 3. The Q^2-dependence of the form factor $G_M/(\mu_\Lambda G_D)$ (a) and G_E (b). The lines correspond the threshold $s_0 = 2.45,\ 2.55,\ 2.65\ \mathrm{GeV}^2$ from up down.

Fig. 4. Contributions of different twist for $G_M(Q^2)$ at the threshold $s_0 = 2.55\ \mathrm{GeV}^2$. The dotted line, dashed line and the solid line correspond to twist 3, twist 4 and all contributions, respectively.

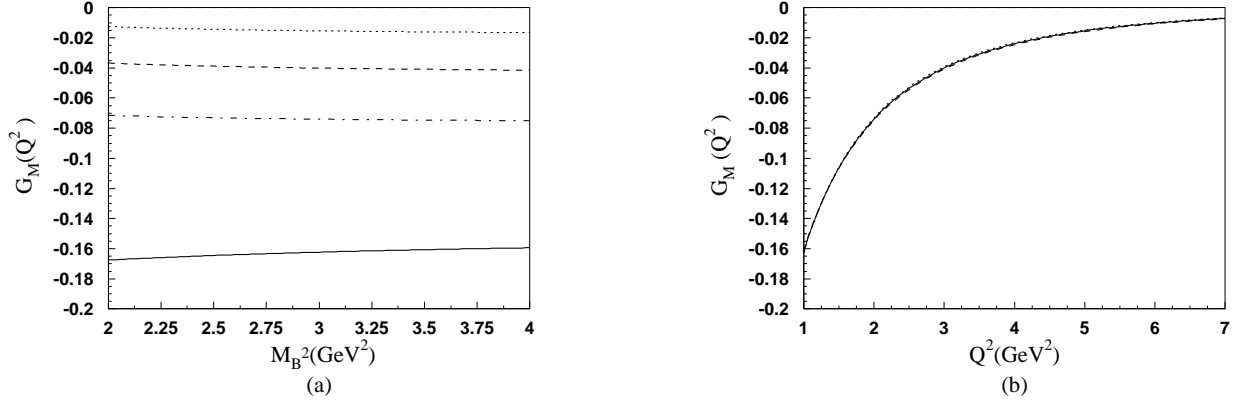Tab. I. The magnetic moment of the Λ baryon from various models.
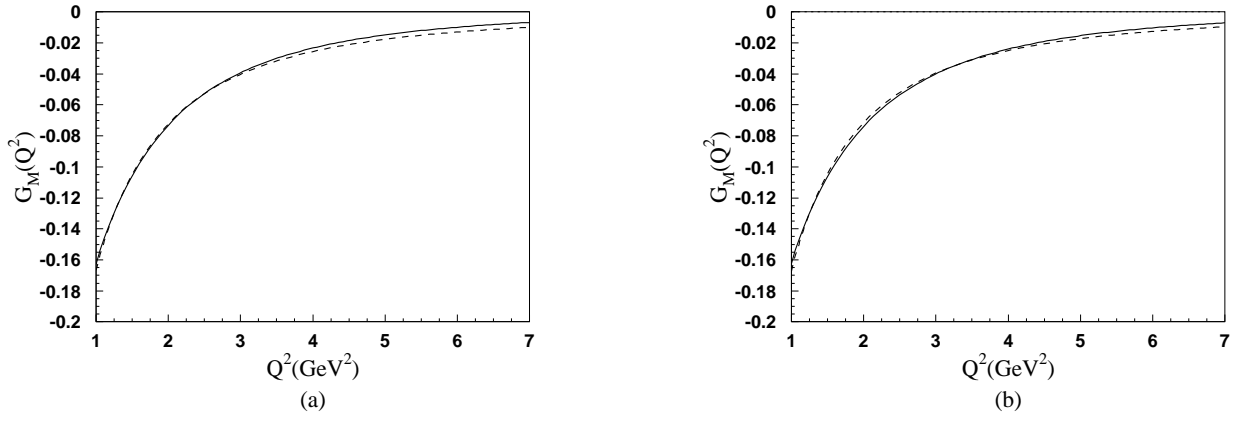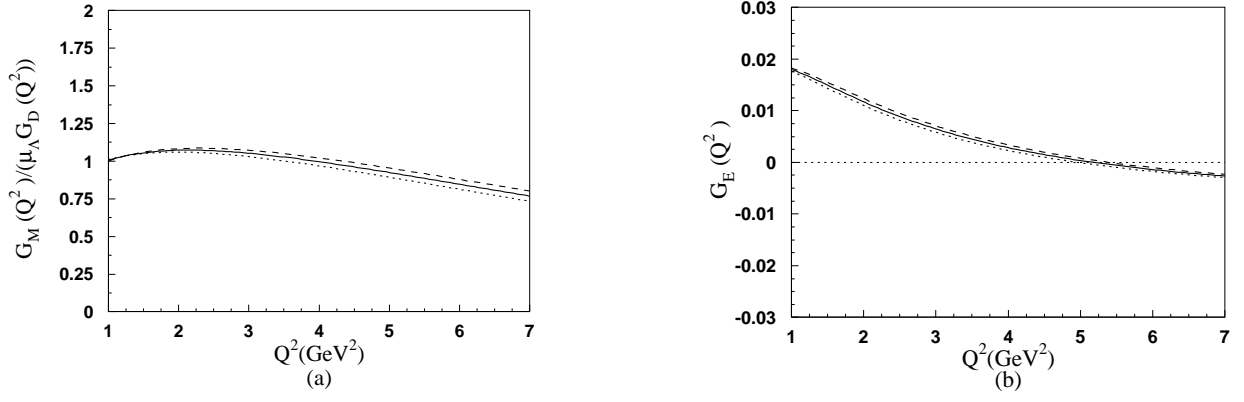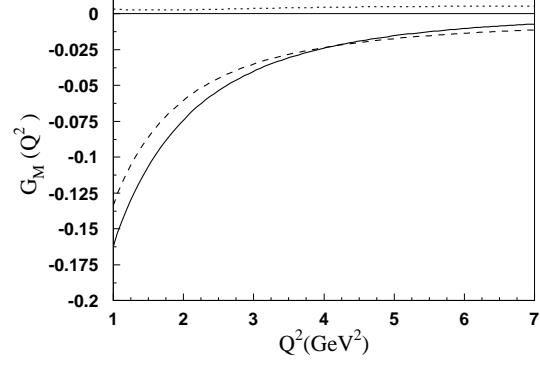
FIG. 1:

FIG. 2:

FIG. 3:

FIG. 4:

TABLE I:

Model	PDG [44]	QCDSR [23]	SQM [24]	QCDSA [25]	χPT [26]	SKRM [27]	NQM [28]	EQLA [29]	GSE [30]	LCSR [31]
$\mu_\Lambda(\mu_N)$	-0.613	$-0.50/$ -0.54	-0.67	-0.69	-0.613	-0.60	-0.63	-0.60	-0.606	-0.7